N·MAHATHI

Pre-Regional Mathematics
Olympiad 2018-19

1. No. of volumes = 3
No. of pages in first volume $= x$
No. of pages in 2^{nd} volume $= x+50$
No. of pages in 3^{rd} volume $= (x+50)\frac{3}{2}$

Page no. of first page in
volume 1 = 1

Page no. of first page in
volume 2 $= x+1$

Page no. of first page in
volume 3 $= (x)+(x+50)+1$
$= 2x+51$

(Given) Sum of 1st page no.s of volume 1, 2 & 3 = 1709.

$\Rightarrow 1 + x+1 + 2x+51 = 1709$

$\Rightarrow 3x + 53 = 1709$

$\Rightarrow 3x = 1656$

$\Rightarrow x = 552$

To find:-

Given ~~on~~ and n is the last page no. of volume 3 we need to find the largest prime factor of n.

n = ~~sum of the~~ the total no. of pages.

$$= x + x + 50 + (x+50)\frac{3}{2}$$

$$= x + (x+50)\frac{5}{2}$$

$$= 552 + \cancel{5}\left(602 \times \frac{5}{2}\right)$$

$$= 552 + \cancel{5}\, 6020/4$$

$= 552 + 1505$

$= 2057$

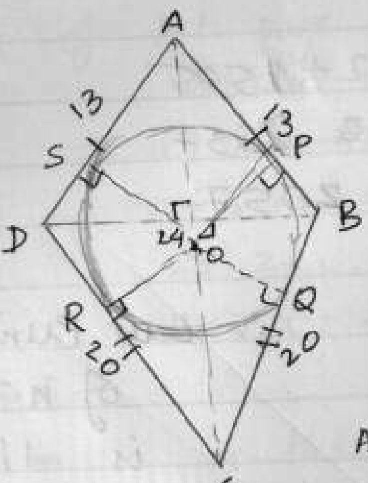
$\Rightarrow n = 2057$

$$\begin{array}{r|l} 11 & 2057 \\ \hline 11 & 187 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

∴ Prime factorisation of $n = 2057$ is $11^2 \times 17$

∴ Largest prime factor of $n = \boxed{17}$

2.



Given - $AB = AD$ & $BC = CD$

$\Rightarrow$ Quadrilateral ABCD is a 'kite'.

$\Rightarrow AC \perp BD$

Also given is circle centered at O of radius 'r', inscribed in ABCD.

We can use properties of congruent triangles in order to ~~prope~~ prove that O lies on AC.

Let us name the points where the circle ~~intersects~~ intersects the quad.. as P, Q, R, S.

$\Rightarrow$ AB, BC, CD, DA are tangent to circle at points P, Q, R, S.

Using properties of tangents to a circle [which can be proved using congruency of triangles] we know that

$PB = BQ$ — ①

$QC = RC$ — ②

$RD = SD$ — ③

$AS = AP$ — ④

~~⇒ AB = BQ + AS~~
~~BC = RC + SD~~

We also know that

$DR = BQ$

$RC = QC$

$DS = BP$

$AS = AP$

- - - (To be cont.d)

6. Given

$a+b-c=1 \quad -(1)$

$a^2+b^2-c^2=-1 \quad -(2)$

Reqd

sum of all possible values of $a^2+b^2+c^2$

From eq $\cancel{2}$ (1)

$(a+b-c)^2 = 1$

$\Rightarrow a^2+b^2+c^2+2ab-2bc-2ac=1$

Since eq (2) $\Rightarrow c^2-1+c^2+2ab-2bc-2ac=1$

$\Rightarrow c^2+ab-bc-ac=1$

Since eq (1) $\Rightarrow ab-c[a+b-1)=1 \quad -(3)$

$\Rightarrow ab-c^2=1$

$\Rightarrow ab=1+c^2$

$\Rightarrow c^2=ab-1 \quad -(4)$

~~But~~ | since eq (1)

$\cancel{(a+b+c)^2=1}$ $\Rightarrow c=\cancel{a+b-1} \quad -(5)$

$\Rightarrow \cancel{a+b=ab} \quad -(6)$

(To be cont.d)

$a+b=1+c$ — From eq ①

$\Rightarrow a^2+b^2-c^2=-1=1+2c-2ab$ [By squaring both sides]

$\Rightarrow -\cancel{2}=\cancel{2}c-\cancel{2}ab$

$\Rightarrow -1=c-ab$

$\Rightarrow ab=c+1$

$\Rightarrow ab=a+b$ — From eq ①

$\Rightarrow ab-a-b+1=1$

$\Rightarrow (a-1)(b-1)=1$

$\Rightarrow$ 2 possible cases

$a-1=1$, $b-1=1$ or $a-1=-1$, $b-1=-1$

$\Rightarrow a=2, b=2$ $\Rightarrow a=0, b=0$

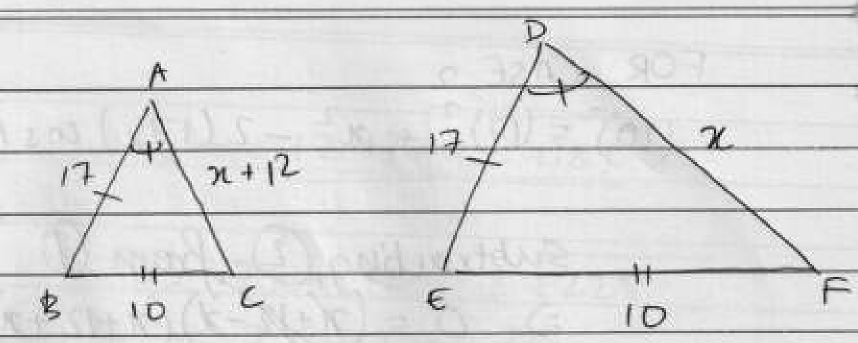
$\Rightarrow c=3$ $\Rightarrow c=-1$

$\Rightarrow$ 2 possible values of $a^2+b^2+c^2$ are

i) 17 ii) 1

$\Rightarrow$ Sum all possible values of $a^2+b^2+c^2=\boxed{18}$

17.



Req.d $AC + DF = 2x+12$

Using cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos(A_o)$$

Here -

	CASE 1	CASE 2
	$a = 10$	$a = 10$
	$b = 17$	$b = 17$
	$c = 12 + x$	$c = x$
	$A_o = A$	$A_o = D = A$

For CASE 1

$$\Rightarrow (10)^2 = (17)^2 + (x+12)^2 - 2(17)(x+12)\cos A \quad -(1)$$

FOR CASE 2

$(10)^2 = (17)^2 + x^2 - 2(17x)\cos A$ — (2)

Subtracting (2) from (1)

$\Rightarrow 0 = (\cancel{x+12-x})(x+12+x) - 2\times 17\cos A[\cancel{x+12-x}]$

$\Rightarrow 2x+12 = 2\times 17\cos A$

$\Rightarrow \frac{2x+12}{34} = \cos A$ — (3)

Substituting (3) in (2)

$\Rightarrow (10)^2 = (17)^2 + (x)^2 - [(x)(2x+12)]$

$\Rightarrow -(x^2 - 2x^2 - 12x) = 7[27]$

$\Rightarrow x^2 + 12x - 189 = 0$

$\Rightarrow x = \frac{-12 \pm \sqrt{144 + 4[189]}}{2}$

Since x cannot be negative

$$x = \frac{-12 + 2\sqrt{36+189}}{2}$$

$$\Rightarrow 2x + 12 = 2\sqrt{225}$$

$$\Rightarrow 2x + 12 = 30.$$

$$\Rightarrow DF + AC = \boxed{30}$$

16) Req. d —
value of

$$\sum_{\substack{1 \le i < j \le 10 \\ i+j = \text{odd}}} (i+j) - \sum_{\substack{1 \le i < j \le 10 \\ i+j = \text{even}}} (i+j)$$

$i + j$ odd

$\Rightarrow (i \text{ is odd}) \cancel{\wedge} \vee (j \text{ is odd})$

But also reqd is $i < j$

If i is odd

$\Rightarrow$

$i =$	1	3	5	7	9
$j =$	2, 4, 6, 8, 10	4, 6, 8, 10	6, 8, 10	8, 10	10

If ~~j~~ i is even ($\Rightarrow j$ odd)

$\Rightarrow$

$i =$	2	4	6	8	10
$j =$	3, 5, 7, 9	5, 7, 9	7, 9	9	-

$$\Rightarrow \sum_{\substack{1 \le i < j \le 10 \\ i + j = \text{odd}}} = 5(1) + 5(2) + 5(3) + \cdots\cdots + 5(9) + 5(10)$$

$$= 5\,[1 + 2 \ldots + 9 + 10]$$

$$= 5 \times \frac{\cancel{10}^{\,5} \times 11}{\cancel{2}_{\,1}}$$

$$= 5 \times 55$$

$i+j$ eve

$\Rightarrow ([i \text{ is odd}] \wedge [j \text{ is odd}])$

$\vee ([i \text{ is even}] \wedge [j \text{ is even}])$

If i is odd

$\Rightarrow i = 1 \vee 3 \vee 5 \vee 7 \vee 9$

$j = 3,5,7,9 \quad 5,7,9 \quad 7,9 \quad 9 \quad -$

If i is even

$\Rightarrow i = 2 \vee 4 \vee 6 \vee 8 \vee 10$

$j = 4,6,8,10 \quad 6,8,10 \quad 8,10 \quad 10 \quad -$

$$\Rightarrow \sum_{\substack{k \le i < j \le 10 \\ i+j = even}} = 4(1) + 4(2) + 4(3) \dots + 4(9) + 4(10)$$

$$= 4[1+2 \dots 9+10]$$

$$= 4 \times \frac{\cancel{10}^{5} \times 11}{\cancel{2}_{1}}$$

$$= 4 \times 55$$

$$\Rightarrow \sum_{\substack{1 \le i < j \le 10 \\ i+j = odd}} - \sum_{\substack{1 \le i \le j \le 10 \\ i+j = even}} = 55(5-4) = \boxed{55}$$

Q2 [cont.d]

$$ar(ABCD) = ar(AOB) + ar[BOC] + ar[COD] + ar[AOD]$$

$$= \frac{1}{2}\, 13 \times x + \frac{1}{2}\, 20 \times x + \frac{1}{2}\, 13 \times x + \frac{1}{2}\, 20 \times x$$

$$= 13x + 20x$$

$$= 43x$$

$$ar[ABCD) = ar(ABD) + ar[BCD)$$

Using heron's formula

$$= \sqrt{25(1)(12)(12)} + \sqrt{32(8)\, 12 \times 12}$$

$$s_1 = \frac{24 + (13)2}{2} = 25$$

$$s_2 = \frac{24 + (20)2}{2} = 32$$

$$= 5 \times 12 + 12 \times 8 \times 2$$

$$= 60 + 160 + 32$$

$$= 252$$

$$\Rightarrow 43x = 252$$

$$\Rightarrow x = \frac{252}{43}$$

$$\sim 5.86$$

$\therefore$ closest integer $= \boxed{6}$

3.

3. Given - 6-digit number
abccba, b is odd
Req.d - no. of 6-digit no
them divisible by 7

$abccba = a + 10b + 100c + 1000c$
$+ 10000b +$
$100000a$

Removing terms divisible by 7
$= a + 3b + 2c + 6c + 4b + 5a$

Removing terms divisible by 7
$= 6a + 7b + 8c$

Removing terms divisible by 7
$= c - a$

$\Rightarrow c - a$ has to be divisible by 7.

Since c, a are digits

$\Rightarrow \boxed{c-a=7}$ or $\boxed{c-a=-7}$

$\Rightarrow (c,a) = (9,2),(8,1),(7,0)$ or $(c,a) = (2,9)(1,8)(0,7)$

or $\boxed{c-a=0 \Rightarrow c=a}$

$\therefore$ No. of possibilities where abccba is divisible by 7, b is odd $=$ ~~28~~ $\boxed{70}$ //

4. $166 = 6 + 6b + \cancel{1}b^2$

$56 = 6 + 5b$

$8590 = 9b + 5b^2 + 8b^3$

$\Rightarrow 30b^3 + 36b^2 + 30b^2 + 36b + 30b + 36 = 8b^3 + 5b^2 + 9b$

$\Rightarrow 30b^3 + 66b^2 + 66b + 36 = 8b^3 + 5b^2 + 9b$

$3b^3 + -61b^2 - 57b - 36 = 0$

4.

4. $16_6 = x^2 + 6x + 6$ where $x = b$

$5_6 = 5x + 6$

$859_0 = 8x^3 + 5x^2 + 9x$

$\Rightarrow [x^2 + 6x + 6][5x + 6] = 8x^3 + 5x^2 + 9x$

$\Rightarrow 5x^3 + 6x^2 + 30x^2 + 36x + 30x + 36 = 8x^3 + 5x^2 + 9x$

$\Rightarrow 5x^3 + 36x^2 + 66x + 36 = 8x^3 + 5x^2 + 9x$

$\Rightarrow 3x^3 - 31x^2 - 57x - 36 = 0$

If α, β, γ are the roots

$\alpha \times \beta \times \gamma = \cancel{12}\ \frac{36}{3} = 12$

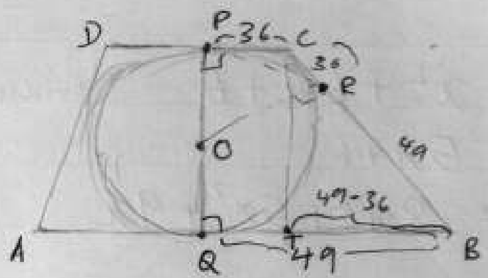
So possibilities for $b \geq 10$ is 12

$3(12)^3 - 31(12)^2 - 57(12) - 36 = 0$

$\therefore$ The only base $b = 12$

$\therefore$ sum of all base b's $= \boxed{12}$

5.



Since $CD \parallel AB$ and $OP \perp CD$ and $OQ \perp AB$

$\Rightarrow$ P, O, Q should lie in a straight line.

$\Rightarrow$ P, O, Q is a diameter of the incircle of $ABCD$.

★ Using properties of tangents

(a) $PC = CR = 36$,

(b) $QB = RB = 49$

$\Rightarrow$ $BC = 36 + 49 = 85$

$\Rightarrow (CT)^2 + [TB]^2 = [BC]^2$ (By pythagoreas theorem)

$TB = QB - QT$

$= QB - CP$

$= 49 - 36$

$= 13$

$CT = \cancel{PQ}\ \overline{PQ}$

$\Rightarrow (PQ)^2 = [BC]^2 - [TB]^2$

$= (85)^2 - (13)^2$

$= (85-13)(85+13)$

$= (72)(98)$

$= 2 \times 6^2 \times 98$

$= 2 \times 6^2 \times 2 \times 7^2$

$\Rightarrow PQ = \sqrt{2^2 \times 6^2 \times 7^2} = 2 \times 6 \times 7$

$= 84$

$\therefore PQ = \boxed{84}$

2 | 72
6 | 36
6 | 6
1

2 | 98
7 | 49
7 | 7
1

9. Let $a+b$ & α be the roots of x^2+ax+b

$\Rightarrow a+b+\alpha = -a$

$\Rightarrow \alpha = -2a-b \quad -(1)$

$(a+b)(\alpha) = b$

$\Rightarrow \alpha = \dfrac{b}{a+b}$

$\Rightarrow (-2a-b)[a+b] = b$

$\Rightarrow -2a^2 - 3ab - b^2 = b$

$\Rightarrow -2a^2 - [3a+1]b - b^2 = 0$

$\Rightarrow 2a^2 + 3ab + b + b^2 = 0$

$\Rightarrow b^2 + (3a+1)b + 2a^2 = 0$

$\Rightarrow b = \dfrac{-3a-1 \pm \sqrt{9a^2+6a+1-8a^2}}{2}$

$= \dfrac{-3a-1 \pm \sqrt{a^2+6a+1}}{2}$

$\Rightarrow a^2+6a+1$ has to be a perfect square.

But since a^2+6a+1 cannot be a perfect square in polynomial terms.

$\Rightarrow a[a+6]+1$ is a perfect square

$\Rightarrow a=0$ or $a+6=0$

$\Rightarrow a=-6$

If $a=0$

$\Rightarrow b = \dfrac{-\cancel{1} \pm \sqrt{1}}{2} = \dfrac{-\cancel{1}\pm 1}{2} = 0$ or -1

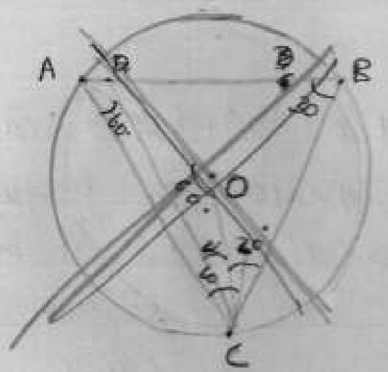
If $a=-6$

$\Rightarrow \cancel{b} = \dfrac{+18-1 \pm \sqrt{1}}{2}$

$= \dfrac{\cancel{+18}+17\pm 1}{2} = \cancel{+10}\ \cancel{9}$
$9, 8$

$\therefore$ Max value of $b^2 = \boxed{81}$

8.



Req.d angle
$\angle ODC$

Given $\angle ABC = 30$

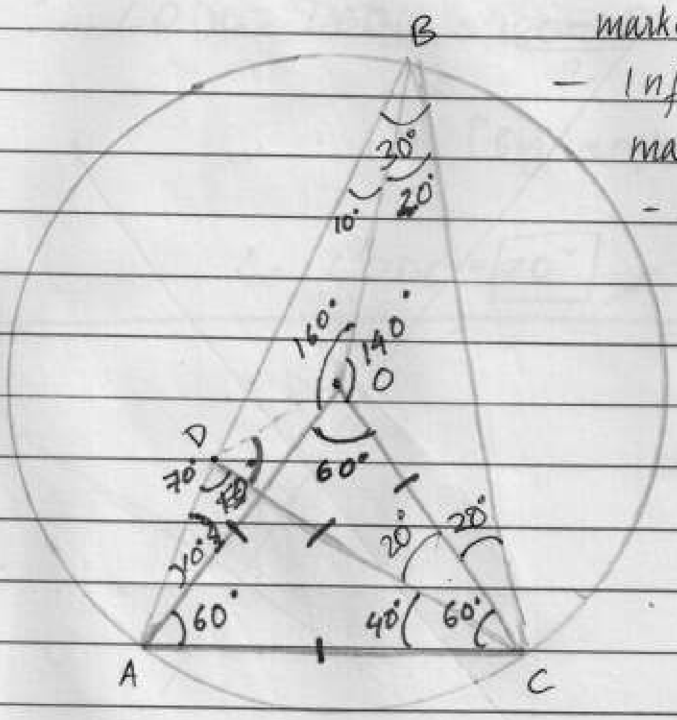
$\Rightarrow \angle AOC = 60^\circ$ [~~Ag~~ angle at center is twice the angle subtended at any point on the circle

$AO = OC$ = radii

$\Rightarrow \angle OAC = \angle OCA = \frac{180-60}{2} = 60^\circ$

$\therefore$ AOC is equilateral

- Given angles marked in black
- Inferred angles marked in blue
- Req.d angles marked in green



$\because$ $\angle OCB$ is 20 $\Rightarrow$ $\angle OAB = 180 - [60+60+20+30]$

$= 10°$

~~By obtaining intermediate angles using~~

$\angle CAD = \angle CDA = 70°$ Angle sum property of triangles

$\therefore \Delta CAD$ is isoceles

$\Rightarrow AC = CD$

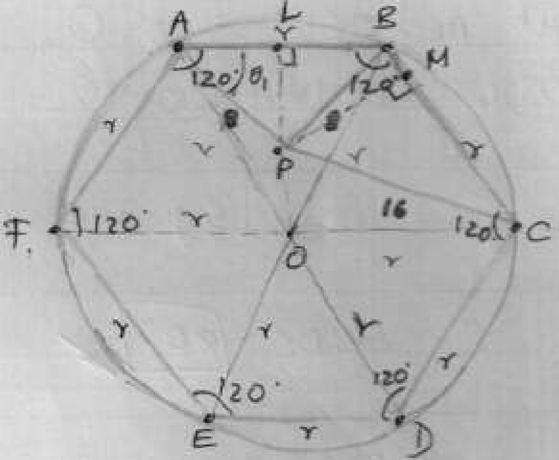
But $AC = OC \Rightarrow CD = O$

$\therefore \angle ODC = \angle COD = \frac{180-20}{2} = 80^{\circ}$

[By anglesum property]

$\therefore \angle ODC = \boxed{80^{\circ}}$

7.



Given –

- $AP = PB = 8$
- $PC = 16$
- ABCDEF is a regular hexagon

$$OL = r\sin 60$$

$$= \frac{\sqrt{3}\,r}{2}$$

$$\cos\theta_1 = \frac{r/2}{8} = \frac{r}{16}$$

$$\Rightarrow \sin\theta_1 = \cancel{\sqrt{64+4r^2}}\ \frac{\sqrt{256-r^2}}{16}$$

$$\Rightarrow \frac{LP}{AP} = \frac{\sqrt{256-r^2}}{16}$$

$$\Rightarrow LP = \frac{\sqrt{256-r^2}}{2}$$

$$\Rightarrow OP = \frac{\sqrt{3}\,r - \sqrt{256-r^2}}{2}$$

Using pythagoreas theorem

$$(OP)^2 + (OC)^2 = (PC)^2$$

$$\Rightarrow \frac{3r^2 + 256 + r^2 - 2\sqrt{768 - 3r^2} + 4r^2}{4}$$

$$= 256$$

$$\Rightarrow \frac{8r^2 + 256 - 2\sqrt{768-3r^2}}{4}$$

$$= 256$$

$$\Rightarrow 8r^2 + 256 - 2\sqrt{768-3r^2} = 1024$$

$$\Rightarrow 8r^2 - 768 = 2\sqrt{768-3r^2}$$

$$\Rightarrow 4r^2 - 384 = \sqrt{768-3r^2}$$

$$\Rightarrow 16r^4 + 147456 - 3072r^2 = 768 - 3r^2$$

$$\Rightarrow 16r^4 - 3069r^2 + 146688 = 0$$

$$\Rightarrow r^2 = \frac{3069 \pm \sqrt{30729}}{32}$$

$$=$$

$$OP^2 = \frac{3r^2 + 256 - r^2 - 2\sqrt{768r^2 - 3r^4}}{4}$$

$$OC^2 = r^2$$

$$PC^2 = 256$$

$$\Rightarrow OP^2 + OC^2 =$$

$$2r^2 + 256 - 2\sqrt{768r^2 - 3r^4} + 4r^2 = 1024$$

$$\Rightarrow 6r^2 + 256 - 1024 = 2\sqrt{768r^2 - 3r^4}$$

$$\Rightarrow 3\cancel{6}r^2 - \overset{384}{\cancel{768}} = \cancel{2}\sqrt{768r^2 - 3r^4}$$

$$\Rightarrow 3r^2 - 384 = \sqrt{768r^2 - 3r^4}$$

$$\Rightarrow 9r^4 - 2304r^2 + 147456 = 768r^2 - 3r^4$$

$$\Rightarrow 12r^4 - 3072r^2 + 147456 = 0$$

$$\Rightarrow r^4 - 256r^2 + 12288 = 0$$

$$r^2 = \frac{256 \pm \sqrt{65536 - 49152}}{4^2}$$

$$= \frac{256 \pm \sqrt{16384}}{4^2}$$

$$= \frac{256 \pm 128}{2}$$

$$= 64 \text{ or } \frac{384}{2} = 192$$

$\therefore r = 8$ or $\sqrt{192}$

~~But since r is given to be an integer~~

~~$\therefore r = \boxed{8}$~~

Req. d is closest integer to r.

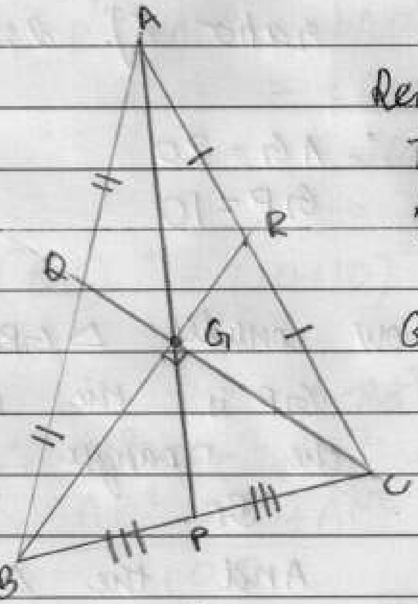
r cannot be 8 as then P will coinside with O

$\Rightarrow 16 \cos C = 8 \Rightarrow 16 = 8$ This not possible

$\therefore\ r = \sqrt{192}$

closest integer to $\sqrt{192}$ is $\boxed{14}$

100



Req.d -
To find
$\frac{AB^2 + BC^2 + AC^2}{100}$

Given -
$CQ \perp BR$
$AP = 30$

We know that G [the centroid] is the point of concurrence of

CQ, AP, BR [The medians of ABC]

We also know that
the centroid of a triangle
divides the medians in
a ratio of 2:1.

$\therefore AG = 20$
$GP = 10$

Now consider ΔGBC.
GP is the median of
the triangle from G to
BC.
And the Δ is right angled
at G.

Then we know that $BP = PC = GP$

$\therefore BP = PC = 10$

By Apollonius Theorem

$$AB^2 + AC^2 = 2AP^2 + 2BP^2$$
$$= 2[30]^2 + 2[10]^2$$
$$= 200[9+1]$$
$$= 2000$$
$$(BC)^2 = (10+10)^2 = (20)^2$$
$$= 2400$$

$$\therefore AB^2 + BC^2 + AC^2 = 2400$$
$$\Rightarrow \frac{AB^2 + BC^2 + AC^2}{100} = \boxed{24}$$

11. Let x = no. of cups with handles

y = no. of cups without handles

$\Rightarrow \cancel{x[x-1] + y[y-1][y-2] = 1200}$

$\Rightarrow \cancel{x^2 - x + y^3 - 3y^2 + 2y = 1200}$

$$\Rightarrow \frac{x[x-1][y-1][y-2]y}{2 \times 6} = 1200$$

$$\Rightarrow x[y][x-1][y-1][y-2] = 12 \times 12 \times 25 \times 4$$

$$= 2^6 \times 3^2 \times 5^2$$

$\therefore$ Possible cases for x, y satisfying above case are

a) $y = 4$, $x = 25$

i.e $25 \times 4 \times 24 \times 3 \times 2$

b) $y = 5$, $x = 16$

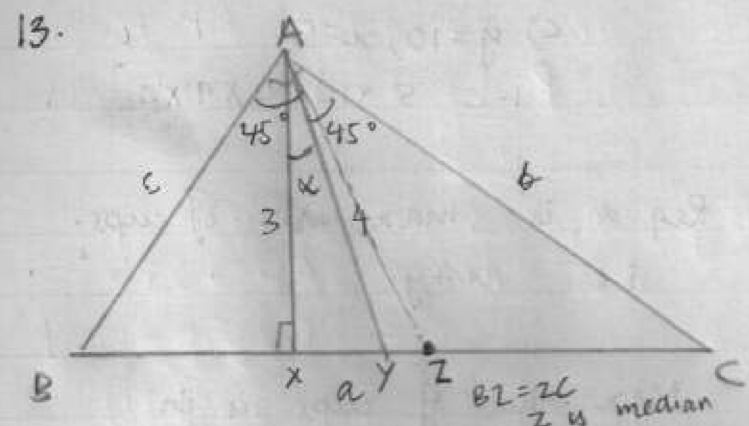
i.e $16 \times 5 \times 15 \times 4 \times 3$

c) $y = 10, x = 5$
i.e $5 \times 10 \times 4 \times 9 \times 8$

Req.d is max. no. of cups.
i.e $x + y$

Max no. of cups is in
case (a) $= \boxed{29}$

13.



Since AY is the angle bisector of $\angle A$.

$\angle BAY = \angle CAY = 45^\circ$

$\Rightarrow \angle C + \alpha = 45^\circ$ [$\because \triangle ABC \sim \triangle XBA$]

And using pythagoras theorem

$XY = \sqrt{4^2 - 3^2}$

$= \sqrt{16-9} = \sqrt{7}$

$\Rightarrow \tan \alpha = \dfrac{\sqrt{7}}{3}$

In $\triangle$ AXC

$$\Rightarrow \tan[45+\alpha] = \frac{\tan 45 + \tan\alpha}{1 - \tan 45 \tan\alpha}$$

$$= \frac{1 + \sqrt{7}/3}{1 - \sqrt{7}/3}$$

$$= \frac{3+\sqrt{7}}{3-\sqrt{7}}$$

$$\Rightarrow \frac{XC}{3} = \frac{3+\sqrt{7}}{3-\sqrt{7}}$$

$$\Rightarrow XC = \frac{9+3\sqrt{7}}{3-\sqrt{7}}$$

In $\triangle$ AXB

$$\tan[45-\alpha] = \frac{\tan 45 - \tan\alpha}{1 + \tan 45 \tan\alpha}$$

$$= \frac{1 - \sqrt{7}/3}{1 + \sqrt{7}/3}$$

$$= \frac{3-\sqrt{7}}{3+\sqrt{7}}$$

$$\Rightarrow \frac{BX}{3} = \frac{3-\sqrt{7}}{3+\sqrt{7}}$$

$$\Rightarrow BX = \frac{9-3\sqrt{7}}{3+\sqrt{7}}$$

$$\Rightarrow BC = \frac{9+3\sqrt{7}}{3-\sqrt{7}} + \frac{9-3\sqrt{7}}{3+\sqrt{7}}$$

$$= 3\left[\frac{9+\cancel{6\sqrt{7}}+7+9-\cancel{6\sqrt{7}}+7}{9-7}\right]$$

$$= 3\left[\frac{18+14}{2}\right]$$

$$= 3[9+7]$$

$$= 48$$

$$\Rightarrow BZ = CZ = 24$$

In a right triangle we know that median from vertex containing right angle to the opposite side is equal to half the length of the opp side.

$\Rightarrow AZ = BZ = CZ$

$\Rightarrow AZ = 24$

$\therefore$ Length of median through $\angle A = \boxed{24}$

14. Using $\cos\theta = \sin[90-\theta]$

$$\Rightarrow x = \cos 1^\circ \cos 2^\circ \cos 3 \dots \cos 89^\circ$$

$$= \cos 1^\circ \sin 1^\circ \cos 2 \sin 2^\circ \dots \cos 44^\circ \sin 44 \cos 45$$

We know that $2 \cos A \sin A = \sin 2A$

$$\Rightarrow x = \frac{2^{44}}{2^{44}} \cos 1 \sin 1 \dots \cos 45$$

$$= \frac{1}{2^{44}} \times \sin 2 \sin 4 \sin 6 \sin 8 \dots \sin 88 \times \cos 45$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2^{44}} \times \sin 2 \sin 4 \sin 6 \dots \sin 88$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2^{44}} \times \sin 2 \sin 88 \sin 4 \sin 86 \ldots\ldots$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2^{44}} \times \sin 2 \cos 2 \sin 4 \cos 4 \ldots \sin 44 \cos 44$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2^{44}} \times \frac{1}{2^{22}} \times \sin 4 \sin 8 \ldots \sin 88$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2^{66}} \times \cos 86 \times \cos 82 \ldots \cos 2$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2^{66}} \times y$$

$$\Rightarrow x = \frac{1}{\sqrt{2}} \times \frac{1}{2^{66}} \times y$$

$$\Rightarrow \frac{y}{x} = 2^{66} \times \sqrt{2}$$
$$= 2^{66.5}$$

$$\Rightarrow \log_2(y/x) = \log_2(2^{66.5})$$

$$= 66.5$$

$$\Rightarrow \frac{2}{7} \times \log_2[y/x] = 66.5 \times \frac{2}{7}$$

$$= \frac{133}{2} \times \frac{2}{7}$$

$$= 19$$

$$\therefore \frac{2}{7} \log_2(y/x) = \boxed{19}$$

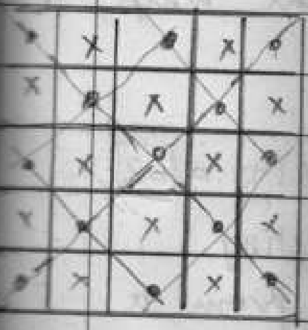
26. Unde

"n

t

grou

i)

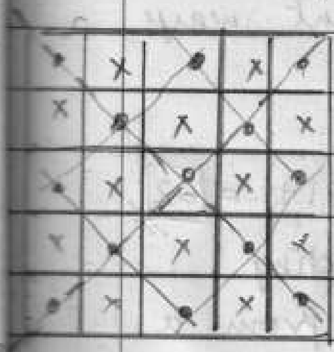


For visualisation ii)

Dots indicate case II, Crosses indicate case 1.

w

26. Under the conditions that
"no ~~two~~ chosen squares have a side
in common"
there are 2 ways of
grouping the squares
i) containing all the square
not on the diagonal ~~not~~
& not along it.
This case contains 60 squares
as req.d. This done only
in 1 unique combination



For visualisation
Dots indicate
case II,
crosses indicate
case I.

ii) containg all the squares
on the diagonal and along
it.
∵ the chessboard is 11x11
where 11 is odd this case

contains 61 squares.

Out of these 61 we need to choose 60 squares.
=> We need to leave out 1 one square.
There are 61 different ways to do this.

∴ Totally there are $61+1 = \boxed{62}$ way of selecting 60 squares from a 11×11 chessboard with the given condition.

28.

dist
Suc
one
wa

Sin
di

28. 8 chocolates can be distributed amongst 3 children such each child gets atleast one chocolate in the following ways.

$\rightarrow 1, 3, 4$

$\rightarrow 1, 2, 5$

Since all their brands are different and also the children

$$N = \left(\frac{8!}{1! \times \cancel{3!} \times 4!} \times \cancel{3!}\right) + \left[\frac{8! \times 3!}{1! \times 2! \times 5!}\right]$$

For case I

For case II

$$= 8 \times 7 \times 6 \times 5 + 6 \times 7 \times 8 \times 3$$

$$= 8 \times 7 \times 6 \, [8]$$

$$= 64 \times 42$$

$= 2560 + 128$

$= 2688$

$\therefore$ sum of the digits of

$N = 2+6+8+8$

$= \boxed{24}$

---

30. In $a_0 + a_1 x + a_2 x^2 \cdots a_n x^n$

where

a_i is ~~positive~~ non negative integer for

$\forall\, i \in \{0, 1, 2, 3 \ldots n\}$

given

$p(1) = 4$

$\Rightarrow a_0 + a_1 + a_2 \ldots a_n = 4$

p(5

We

1 10 125 136

$$p(5) = a_0 + 5a_1 + 25a_2 + 125a_3$$
$$\ldots\ldots + 5^n a_n = 136$$

We notice the following things

* Maximum of 4 a_i's can be non-zero.
* No a_i can be greater than 4.
* Since when 136 is divided by 4 = 34 which is not a ~~multiple~~ Power of 5, a_i cannot be equal to 4.
* Now considering possible sums to equal 4

i) 1, 1, 2
ii) 1, 3
iii) 2, 2
iv) 1, 1, 1, 1

* Terms from a_3 to a_n are zero since all of them are > 136

∴ By ~~rather~~ trial and error

a_0, a_1, a_2, a_3

$= 1, 2, 0, 1$

$\therefore p(3) = a_0 + 3a_1 + 27a_3$

$= 1 + 6 + 27$

$= \boxed{34}$ //